CALCULATION OF ELECTRON ENERGY LOSSES IN VARIOUS METALS

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in Various Metals

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ABSTRACT

A reformulation of the Blunck and Westphal theory of electron energy loss in metals was performed for the metals beryllium, aluminum, copper, tin, gadolinium, and lead. Comparison with previous theoretical calculations shows good agreement for the most probable energy loss and for the full widths at half maximum of the electron energy loss distributions. The computer program designed for these calculations is an improvement over previous programs in both computation time and simplicity.

A semi-smpirical formula for the most probable energy loss was calculated. This formula agrees, within a few percent, with the most probable energy loss calculated according to the Blunck and Westphal theory except in the case of thick (>3 gm/cm^2) absorbers of heavy elements. The full width at half maximum for the energy loss distribution is presented in graphical form as functions of target thickness and atomic number. Both the most probable energy loss and the half widths were found to be only slightly dependent upon the initial electron energy.

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I. INTRODUCTION

The theory of energy loss of monoenergetic electrons was first treated by Landau in 1944 [1]; revised by Blunck and Leisegang in 1950 [2]; and Blunck and Westphal in 1951 [3]. The Blunck and Westphal theory assumes an incident beam of monoenergetic electrons and treats energy losses due to both radiation (bremsstrahlung) and to ionization/excitation of atomic electrons. In the theory, energy transferred to the recoil nucleus is neglected.

Previously, experimental studies of the energy loss of high energy electrons in metals have been performed at the Naval Postgraduate School by Bumiller, Buskirk, Dyer, and Miller [4], Miller [5], Goodwin [6], Deleuil and Raynis [7], Mosbrooker and Sandquist [8], and by Barry and Oppedahl [9]. Comparison of theoretical and experimental values, for the most probable energy loss and for the full width at half maximum for the energy loss distribution, have given consistent and comparable results for thin targets. However, previous computer programs designed to numerically compute the energy loss distribution have encountered divergence problems in certain integrals, and have been unsuccessful in the treatment of data from experiments with targets of high atomic number and large thickness. K. Whoeler [10] has reformulated the expressions in the Blunck and Westphal theory so that tractable numerical calculations can be achieved. The energy loss distributions of electrons have been recalculated using this reformulation and satisfactory comparison to experiment and previous calculations has been achieved with a significant reduction in computation time.

An important parameter in all electron energy loss experiments is the value of the most probable energy loss for a given thickness of target. A semi-empirical formula has been determined which will give the correct value of the most probable energy loss to within about 3% for all absorbers except ones made of thick, heavy elements. This formula is a function of the atomic number, the atomic weight, the target thickness in gm/cm^2 and the density in gm/cm^2. The full width at half maximum of the energy loss distribution is also an important parameter to the experimenter. A graphical presentation of the half-widths of the energy loss distributions is given in order to aid the experimenter in anticipating the values of the half-width without using the time-consuming computations of the Blunck and Westphal theory.

II. THEORETICAL CONSIDERATIONS

A. THE BLUNCK AND WESTPHAL THEORY

The probability that an electron of energy E_i looses an amount of energy between Q and Q + dQ is W(Q)dQ. The expression for W(Q)dQ is given by Blunck and Westphal [3].

$$W(Q)dQ = \int_{q=0}^{Q}\int W_I(Q-q)W_s(q)dqdQ \qquad (1)$$

where

$W_I(Q-q)dQ$ = probability of energy loss Q-q, due to ionization

$W_s(q)dq$ = probability of energy loss q, due to radiation

$W_I(Q-q)W_s(q)dqdQ$ = joint probability of radiation loss q and ionization loss Q-q.

In this theory it is assumed that the energy loss Q is small compared to the incident beam of energy, E_i, that is, $Q \ll E_i$.

The probability distribution for energy loss due to ionization and excitation is [3]:

$$W_I(Q-q)dQ = \sum_{\gamma=1}^{4} \frac{C_\gamma \gamma_\gamma}{\sqrt{b^2 + \gamma_\gamma^2}} \exp\left[\frac{-(\lambda - \lambda_\gamma)^2}{b^2 + \gamma_\gamma^2}\right] d\lambda \qquad (2)$$

where C_γ, γ_γ and λ_γ are constants evaluated by Blunck and Westphal to fit the above equation to an approximation composed of four Gaussian functions, λ is the Landau lambda, a parameter related to the energy loss Q.

The Landau lambda was defined by Blunck and Leisegang as:

$$\lambda = \frac{Q}{ax} - \frac{\overline{K}}{a} + \ln\left(\frac{E_i}{ax}\right) - C \qquad (3)$$

with C = 1.116.

In the equation for lambda,

Q is the total energy loss traversing a path distance, x,

"a" = $0.154Z/\beta^2\rho A$ = basic cross section constant,

E_i is the initial electron energy, and

$\overline{K}$ is the average energy loss per centimeter of target material.

If one uses $\overline{K}$, the average energy loss per length of target, as derived by Sternheimer [11] and accounts for density effects, C, the numerical constant in the Landau lambda becomes C = 0.686, as shown by Whoeler. A derivation of this constant can be found in Appendix C.

The probability that an electron looses an amount of energy between q and q+dq by radiation alone is according to the Blunck and Westphal theory [3].

$$W_s(q)dq = \frac{1}{\Gamma(z)} \left(\frac{q}{E_i}\right)^z \frac{dq}{q} \qquad (4)$$

where $z = \alpha_R \rho R$

$\alpha_R = 1.4 \times 10^{-3} \frac{Z^2}{A} F$

$F = \frac{4}{3} \ln (183\, Z^{-1/3}) + \frac{1}{9}$.

α_R is equal to the inverse radiation length times the ln 2, and F is a correction constant taking into account complete screening of the nucleus by the orbit electrons. The derivation of these two constants can be found in Appendices D and E. The factor of ln 2 in this expression is a consequence of the theory of radiation loss according to Heitler and Bethe [12], in which the radiation length is defined as that length where the electron has lost half of its energy to radiation.

Combination of equations 2 and 4 gives the expression for the total probability, W(Q)dQ. If the following is defined:

$$\phi = \frac{q}{aX} \tag{5}$$

$$\Phi(\lambda - \phi) = \sum_{\gamma} \frac{C_\gamma \gamma_\gamma}{\sqrt{b^2 + \gamma_\gamma^2}} \exp\left[\frac{-(\lambda - \lambda_\gamma - \phi)^2}{b^2 + \gamma_\gamma^2}\right] \tag{6}$$

it is possible to obtain using expressions (1) and (2) the following:

$$W(Q)dQ = \int_0^Q \Phi(\lambda-\phi) \frac{1}{\Gamma(z)} \left(\frac{aX}{E_i}\right)^z \phi^z \frac{d\phi}{\phi} \, d\lambda \tag{7}$$

$$= \left(\frac{aX}{E_i}\right)^z \frac{1}{\Gamma(z)} \sum_{\gamma} \frac{C_\gamma \gamma_\gamma}{\sqrt{b^2 + \gamma_\gamma^2}} \left[\int_0^Q \exp\left[\frac{-(\lambda-\lambda_\gamma-\phi)^2}{b^2 + \gamma_\gamma^2}\right] \phi^{z-1} \, d\phi\right] d\lambda \, .$$

With the transformation $\phi = \frac{\eta}{2}\sqrt{b^2 + \gamma_\gamma^2}$, it is possible to define a new integration variable η, so that the total probability becomes:

$$W(Q)dQ = \left(\frac{aX}{E_i}\right)^z \left(\frac{1}{2}\right)^{z/2} \frac{1}{\Gamma(z)} \sum_{\gamma} \frac{C_\gamma \gamma_\gamma \, e^{\frac{-(\lambda-\lambda_\gamma)}{b^2 + \gamma_\gamma^2}}}{\sqrt{(b^2 + \gamma_\gamma^2)^{1-z}}}$$

$$\left[\int_0^{\eta_m} \eta^{z-1} \exp\left(\frac{\sqrt{2}\,(\lambda-\lambda_\gamma)}{b^2 + \gamma_\gamma^2} \eta - \frac{\eta^2}{2}\right) d\eta\right] d\lambda \, . \tag{8}$$

The variable q in equation (1) is related to the variable, η, and Q is related to λ. For convenience, Λ_γ is defined as:

$$\Lambda_\gamma = \frac{\lambda - \lambda_\gamma}{\sqrt{b^2 + \gamma_\gamma^2}} \, ,$$

in order to express equation (8) in an apparently simpler form,

$$W(Q)dQ = \left(\frac{aX}{E_i}\right)^z \left(\frac{1}{2}\right)^{z/2} \frac{1}{\Gamma(z)} \sum_{\gamma} \frac{C_\gamma \gamma_\gamma}{\sqrt{(b^2 + \gamma_\gamma^2)^{1-z}}} \int_0^{\eta_m} \eta^{z-1} e^{\sqrt{2}\,\Lambda_\gamma \eta - \frac{\eta^2}{2}} \, d\eta \, d\lambda \, . \tag{9}$$

It is found that for values of z less than 1.0, that is, for relatively thin targets, the integrand tends to infinity at the lower limit of integration and numerical evaluation of the integral in this form is not possible. Transformation of the above equation by series expansion and integration by parts will produce a form that does not diverge and produces an integral that can be computed numerically (see Appendix F for derivation of removal of divergence). This procedure will produce the following equation:

$$W(Q)dQ = \left(\frac{aX}{E_i}\right)^z \left(\frac{1}{2}\right)^{z/2} \frac{1}{\Gamma(z)} \sum_\gamma \frac{C_\gamma \gamma_\gamma}{\left(b^2 + \gamma_\gamma^2\right)^{1-z}} \left[\frac{1}{z} \eta_m^2 \exp\left(\sqrt{2}\Lambda\eta_m - \frac{\eta_m^2}{2}\right) - \int_0^{\eta_m} \frac{1}{z} \eta^z \exp\left(\sqrt{2}\Lambda_\gamma \eta - \frac{\eta^2}{2}\right) d\eta \right] d\lambda \quad . \tag{10}$$

Earlier, z was defined as $z = \alpha_R \rho X$. This is also equal to $\alpha_R T$ where T is the target thickness in gm/cm^2. Finally by defining $\tau = z \ln 2 = \alpha_R T \ln 2$, the final expression used in the calculation becomes:

$$W(Q)dQ = \left(\frac{1}{\tilde{a}T}\right)\left(\frac{\tilde{a}T}{E_i}\right)^\tau \frac{1}{2}^{\tau/2} \frac{1}{\Gamma(\tau)} \sum_{\gamma=1}^{4} \frac{C_\gamma \gamma_\gamma e^{-\Lambda_\gamma^2}}{\sqrt{\left(b^2 + \gamma_\gamma^2\right)^{1-\tau}}} T_\gamma \tag{11}$$

where:

$$\tilde{a} = \frac{a}{\rho} = \frac{0.154\ Z}{\beta^2 A}$$

and,

$$T_\gamma = \frac{1}{\tau} (\eta_n)^\tau \exp\left[\Psi(\eta_m)\right] - \int_0^{\eta_m} \frac{1}{\tau} \eta^\tau \left[\sqrt{2}\Lambda_\gamma - \eta\right] \exp\left[\Psi(\eta)\right] d\eta \cdot$$

and,

$$\Psi(\eta) = \sqrt{2}\ \Lambda_\gamma \eta - \frac{\eta^2}{2} \quad .$$

The expression $\frac{W(Q)dQ}{d\lambda}$ was calculated from expression (11) using the IBM 360 computer. The code is found in Appendix G and the program is in Appendix H.

B. A SEMI-EMPIRICAL FORMULA FOR THE ENERGY LOSS

The most probable energy loss (Q_p) can be obtained from the Blunck and Westphal theory only by the rather tedious calculations outlined above. For that reason, a semi-empirical fit has been made to the Blunck and Westphal results. Hanson, Goldwasser, and Mills [13, 14] give a semi-empirical formula for the most probable energy loss. It has the form:

$$Q_p = a\ t\ \frac{Z}{A}\left[\ln\frac{t}{\rho} + b\right]$$

where:

a = .154 and b = 17.68,

Z is the atomic number,

A is the atomic weight,

ρ is the density of target in gm/cm^2,

t is the target thickness in gm/cm^2.

The formula agrees with the Blunck and Westphal results to within about 15%.

Presented in this paper for the formula of the most probable energy loss is the equation:

$$Q_p = 0.154\ t\ \frac{Z}{A}\left[\ln\frac{t}{\rho} + 20.085\right]$$

By setting the value of a equal to 0.154 (a collection of fundamental constants) and solving for b in order to fit the formula to the Blunk and Westphal results, a better semi-empirical formula was arrived at,

which gives results that are within 3% of the Blunck and Westphal values except for thick targets of heavy elements.

III. ANALYSIS OF RESULTS

The values for the most probable energy loss and the full width at half maximum found in Tables I through VIII were taken from the calculated energy loss distributions of electrons, using Blunck and Westphal's theory. This final energy loss distribution was found using an initial distribution which was not monoenergetic but which had a finite half width. The initial distribution was then folded into the Blunck and Westphal theory using the histogram method as described by Barry and Oppedahl [9]. In previous works however, the initial distribution was approximated by taking its experimental half width and fitting this half width to a Gaussian function. In this thesis, the actual experimental distribution for the incident electrons was used, taken from data of previous experiments [6, 7, 8, & 9]. In Tables I through VIII, Q_p(B&O) refers to the value of the most probable energy loss as calculated by Barry and Oppedahl; Q_p(MID) refers to the most probable energy loss as calculated in this paper, and Q_p(EXP) refers to the experimental value of the most probable energy loss.

The results for most probable energy loss, Q_p, and half widths, H.W., were then plotted against the target thickness for various metals. These results were compared with the theoretical results calculated by Barry and Oppedahl and with the actual experimental values. This comparison in lead can be see in Figures 1 through 6.

The most probable energy loss and half widths for the electron energy loss distribution of various metals were also calculated using monoenergetic initial electron beam. These parameters, Q_p and H.W.,

were then plotted against target thickness and the resulting curves were then fitted to an empirical formula which will calculate the most probable energy loss and half width given the target thickness, the atomic number Z, the atomic weight A, and the density of the target material. These results are found in Figures 7 through 10.

IV. RESULTS AND CONCLUSIONS

The theoretical results for the most probable energy loss, Q_p, and full width at half-maximum, H.W., for the electron energy loss distributions, as calculated from the reformulation of Blunck and Westphal's energy loss theory, are listed in Tables I through VIII. A comparison of these results with Barry and Oppedahl's theoretical results yields values which are essentially the same for the range of target thickness of the metals used. Comparison with experimental values, taken from previous experiments performed at the Naval Postgraduate School, yield results which are excellent for the most probable energy loss, but which predict the correct half-widths of the distributions for only thin targets of thickness < 3 gm/cm^2. These are the same results arrived at in previous papers on this subject. However, the predicted values for Q_p were, in some cases, closer to the experimental value than the theoretical values presented in previous theses. This is believed to be because the actual experimental initial distribution was used in the calculations rather than a Gaussian approximation and because density effects were properly considered by the redefinition of the Landau lambda.

The discrepancies in the half-widths for the thicker, heavier targets is to be expected based on the assumptions made in the Blunck and Westphal theory. It is assumed in this theory that the energy loss in the target is small compared to the energy of the incident beam. For thick targets (> 3 gm/cm^2) of heavy elements, the energy loss in the target is greater than 10% of the incident beam energy.

The most probable energy loss and the half-width of the energy loss distribution is nearly independent of the incident energy (E_i) of the electron for the range of energies used. Figures 1 through 3 plot Q_p versus target thickness for various values of E_i in lead. Similarly, Figures 4 through 6 show that the half-widths are also nearly independent of incident energy for lead. In Tables I through VIII, comparisons of Q_p and half-width for different target thicknesses, show the energy independence in other metals.

A monoenergetic beam was used to calculate the Blunck and Westphal values for the most probable energy loss and the half-width of the energy loss distribution. These values were plotted against target thickness in order to try and fit the curves to simple empirical formulas which could be used in substitution of the tedious Blunck and Westphal computations. For the most probable energy loss, Q_p, a semi-empirical formula given by Hanson, Goldwasser, and Mills was used which has the form:

$$Q_p = a\, t\, \frac{Z}{A} \left[\ln \frac{t}{\rho} + b \right]$$

The constants a and b were determined as follows; a was selected to be equal to 0.154 which is a collection of fundamental constants equal to:

$$\frac{2\pi e^4}{mc^2} N ,$$

where e is the fundamental charge of the electron, m is the mass of the electron, c is the speed of light, and N is Avogadro's number. Using this value for a, b was determined by fitting the semi-empirical formula to the beryllium data for the most probable energy loss as given by the Blunck and Westphal theory. The value of b obtained was 20.085. This

proved to be significantly different from the value of 17.68 given by Hanson et al for the value of b. This form of the semi-empirical formula agreed to within about 3% of the values for Q_p predicted by Blunck and Westphal. The only exceptions were for thick targets of heavy metals such as gadolinium and lead. This is a significant improvement over the values predicted from the formula by Hanson et al which gives results good to only within about 15% for Q_p. The semi-empirical formula for Q_p is plotted against target thickness and is shown in Figures 7 and 8. Also plotted on these graphs are the theoretical Blunck and Westphal values for the various metals. In addition, Table IX gives the values for Q_p as calculated using the Blunck and Westphal theory, the Hanson, Goldwasser, and Mills empirical formula and the empirical formula as computed in this paper. Percent differences between the semi-empirical formulas and the Blunck and Westphal values are also given.

A similar attempt was made to fit the data of half-widths to an empirical formula, but no simple formula could be found. Instead, a graphical presentation of half-widths is given in order to present the experimenter with a satisfactory estimate of the half-width. Figure 9 shows the half-widths as a function of target thickness for the metals used in this study. The strong "Z" dependence is shown as the heavier elements curve sharply upward. In Figure 10, the half-width is plotted against the fractional radiation length the electrons have for the various thicknesses of target. The graphs should enable one to estimate the half-width of the energy loss distribution if the thickness of the target is known for materials up to the atomic number of lead.

APPENDIX A - TABLES

Table I. Comparison of Most Probable Energy Loss, Q_p, for Beryllium Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	Q_p(B&O)	Q_p(MID	Q_p(EXP)
52.89	0.742	0.98	0.99	0.98 ± .02
	1.479	2.04	2.03	2.00 ± .04
	2.209	3.05	3.08	2.96 ± .05
	2.961	4.17	4.15	4.09 ± .08
	3.673	5.29	5.25	5.19 ± .05
	4.415	6.36	6.40	6.24 ± .13
	5.179	7.46	7.52	7.40 ± .08
	5.908	8.69	8.65	8.38 ± .12
74.78	0.738	1.02	1.00	1.01 ± .03
	1.479	2.04	2.04	2.01 ± .04
	2.209	3.11	3.10	3.04 ± .05
	2.941	4.15	4.15	4.15 ± .14
	3.673	5.29	5.15	5.15 ± .06
	4.435	6.39	6.40	6.28 ± .10
	5.179	7.46	7.50	7.42 ± .13
	5.908	8.68	8.65	8.51 ± .09
94.64	0.738	1.02	1.00	1.05 ± .04
	1.479	2.04	2.05	2.06 ± .04
	2.209	3.11	2.99	3.09 ± .04
	2.941	4.15	4.20	4.06 ± .06
	3.673	5.29	5.27	5.19 ± .06
	4.435	6.39	6.40	6.30 ± .12
	5.179	7.61	7.55	7.47 ± .12
	5.908	8.69	8.63	8.63 ± .14

Table II. Comparison of Full Widths at Half Maximum for Beryllium Energy Loss Distribution.

E_i	$T(g/cm^2)$	H.W.(B&O)	H.W.(MID)	H.W.(EXP)
52.89	0.742	0.33	0.36	0.36 ± .02
	1.479	0.53	0.56	0.56 ± .04
	2.209	0.67	0.76	0.77 ± .04
	2.961	0.90	0.96	1.07 ± .07
	3.673	1.17	1.18	1.25 ± .06
	4.415	1.40	1.41	1.46 ± .09
	5.179	1.76	1.70	1.76 ± .10
	5.908	2.09	1.93	2.19 ± .14
74.78	0.738	0.46	0.44	0.47 ± .04
	1.479	0.56	0.60	0.61 ± .04
	2.209	0.74	0.78	0.79 ± .05
	2.941	0.96	0.96	1.15 ± .08
	3.673	1.17	1.20	1.39 ± .06
	4.435	1.46	1.43	1.61 ± .10
	5.179	1.76	1.68	1.83 ± .08
	5.908	2.12	1.96	2.24 ± .11
94.64	0.738	0.58	0.54	0.57 ± .05
	1.479	0.68	0.70	0.66 ± .04
	2.209	0.83	0.88	0.84 ± .06
	2.941	1.06	1.05	1.22 ± .08
	3.673	1.23	1.28	1.35 ± .08
	4.435	1.53	1.50	1.58 ± .12
	5.179	1.82	1.75	1.93 ± .10
	5.908	2.16	1.98	2.29 ± .11

Table III. Comparison of Most Probable Energy Loss, Q_p, for Tin Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	Q_p(B&O)	Q_p(MID)	Q_p(EXP)
52.53	1.485	1.83	1.80	1.78 ± .05
	2.970	3.84	3.85	3.86 ± .15
	4.455	6.56	6.55	6.35 ± .30
75.00	1.485	1.81	1.80	1.77 ± .07
	2.970	3.93	3.90	3.86 ± .15
	4.455	6.56	6.60	6.65 ± .30
94.40	1.485	1.88	1.90	1.87 ± .10
	2.970	3.93	3.98	3.96 ± .20
	4.455	6.56	6.60	7.00 ± 1.0

Table IV. Comparison of Full Widths af Half Maximum for Tin Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	H.W.(B&O)	H.W.(MID)	H.W.(EXP)
52.53	1.485	0.75	0.78	0.78 ± .10
	2.970	3.03	2.68	3.02 ± .40
	4.455	16.70	16.00	11.30 ± 2.0
75.00	1.485	0.73	0.76	0.83 ± .10
	2.970	2.56	2.72	2.96 ± .42
	4.455	16.69	16.15	15.50 ± 2.0
94.40	1.485	0.83	0.95	0.88 ± .15
	2.970	2.63	2.85	2.72 ± .45
	4.455	16.75	16.00	16.20 ± 4.0

Table V. Comparison of Most Probable Energy Loss, Q_p, for Copper Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	Q_p(B&O)	Q_p(MID)	Q_p(EXP)
52.84	0.711	0.92	0.90	0.94
	1.423	1.89	1.84	1.87
	2.134	2.90	2.80	2.87
	2.845	3.87	3.83	3.80
74.76	0.711	0.92	0.90	0.92
	1.423	1.89	1.85	1.91
	2.134	2.90	2.85	2.92
	2.845	3.96	3.85	3.98
94.30	0.711	0.91	0.90	0.95
	1.423	1.94	1.90	1.94
	2.134	2.96	2.90	2.93
	2.825	3.99	3.90	4.00

Table VI. Comparison of Full Widths at Half Maximum for Copper Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	H.W.(B&O)	H.W.(MID)	H.W.(EXP)
52.84	0.711	0.46	0.40	0.52
	1.423	0.69	0.65	0.84
	2.134	1.04	1.00	1.16
	2.845	1.54	1.60	1.75
74.76	0.711	0.56	0.46	0.66
	1.423	0.78	0.70	1.07
	2.134	1.12	1.15	1.43
	2.845	1.61	1.58	1.98
94.30	0.711	0.67	0.62	0.74
	1.423	0.87	0.80	0.99
	2.134	1.22	1.13	1.47
	2.845	1.71	1.48	1.75

Table VII. Comparison of Most Probable Energy Loss, Q_p, for Lead Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	Q_p(B&O)	Q_p(MID)	Q_p(EXP)
53.85	0.706	0.79	0.80	0.81 ± .05
	1.412	1.66	1.88	1.45 ± .08
	2.118	2.55	2.60	2.64 ± .10
	2.825	3.57	3.70	3.66 ± .12
74.74	0.76	0.83	0.84	0.82 ± .05
	1.412	1.68	1.88	1.80 ± .10
	2.118	2.66	2.65	2.59 ± .12
	2.825	3.66	3.70	3.70 ± .15
91.37	0.706	0.90	1.00	0.87 ± .07
	1.412	1.80	2.00	1.77 ± .10
	2.118	2.70	2.73	2.75 ± .15
	2.825	3.69	3.80	3.84 ± .22

Table VIII. Comparison of Full Widths at Half Maximum for Lead Energy Loss Distribution.

E_i(MeV)	T(g/cm^2)	H.W.(B&O)	H.W.(MID)	H.W.(EXP)
53.85	0.706	0.54	0.56	0.60 ± .06
	1.412	0.97	1.10	1.23 ± .12
	2.118	2.11	2.21	2.65 ± .25
	2.825	6.46	6.32	6.26 ± .50
74.74	0.706	0.78	0.75	0.82 ± .05
	1.412	1.23	1.28	1.47 ± .07
	2.118	2.37	2.35	2.24 ± .19
	2.825	6.52	6.43	6.25 ± .85
91.37	0.706	0.93	0.85	0.78 ± .08
	1.412	1.37	1.35	1.52 ± .19
	2.118	2.53	2.48	2.74 ± .64
	2.825	6.95	6.65	8.80 ± .70

Table IX. Comparison of Semi-Empirical Formula Results for Most Probable Energy Loss, Q_p

$t(gm/cm^2)$	Q_p(B&W)	Q_p(HAN)	PERCENT DIFF.	Q_p(MID)	PERCENT DIFF.
BE(Z=4)					
1.0	1.32	1.16	12.1	1.33	0.8
2.0	2.76	2.43	12.0	2.76	0.0
3.0	4.22	3.73	11.6	4.22	0.0
4.0	5.70	5.05	11.4	5.71	0.2
5.0	7.20	6.39	11.0	7.21	0.1
Al(Z=13)					
1.0	1.38	1.24	10.1	1.41	2.2
2.0	2.86	2.58	9.8	2.94	2.8
3.0	4.40	3.96	10.9	4.50	2.3
4.0	6.02	5.37	10.8	6.08	1.0
Cu(Z=29)					
1.0	1.24	1.09	12.1	1.26	1.6
2.0	2.60	2.27	12.7	2.61	0.4
3.0	4.02	3.60	12.9	4.00	0.5
4.0	5.52	4.74	14.1	5.42	1.8
5.0	7.12	6.00	15.7	6.85	3.8
Sn(Z=50)					
1.0	1.16	1.02	12.1	1.17	0.9
2.0	2.44	2.13	12.7	2.44	0.0
3.0	3.84	3.27	14.8	3.74	2.6
4.0	5.58	4.43	20.6	5.06	9.3
Gd(Z=64)					
1.0	1.12	0.98	12.5	1.13	0.9
2.0	2.40	2.04	15.0	2.35	2.1
3.0	3.84	3.14	18.2	3.60	6.2
4.0	5.86	4.26	27.3	4.87	16.9
Pb(Z=82)					
1.0	1.06	0.92	13.2	1.07	0.9
2.0	2.30	1.94	15.6	2.24	2.6
3.0	2.94	2.99	24.1	3.43	12.9
4.0	7.20	4.05	43.7	4.64	35.6

Q_p(B&W) - most probable energy loss as given by Blunck and Westphal.

Q_p(HAN) - most probable energy loss as calculated from the semi-empirical formula by Hanson et al.

Q_p(MID) - most probable energy loss as calculated from the semi-empirical formula presented in this paper.

All percentage differences are with respect to the Blunck and Westphal Theory.

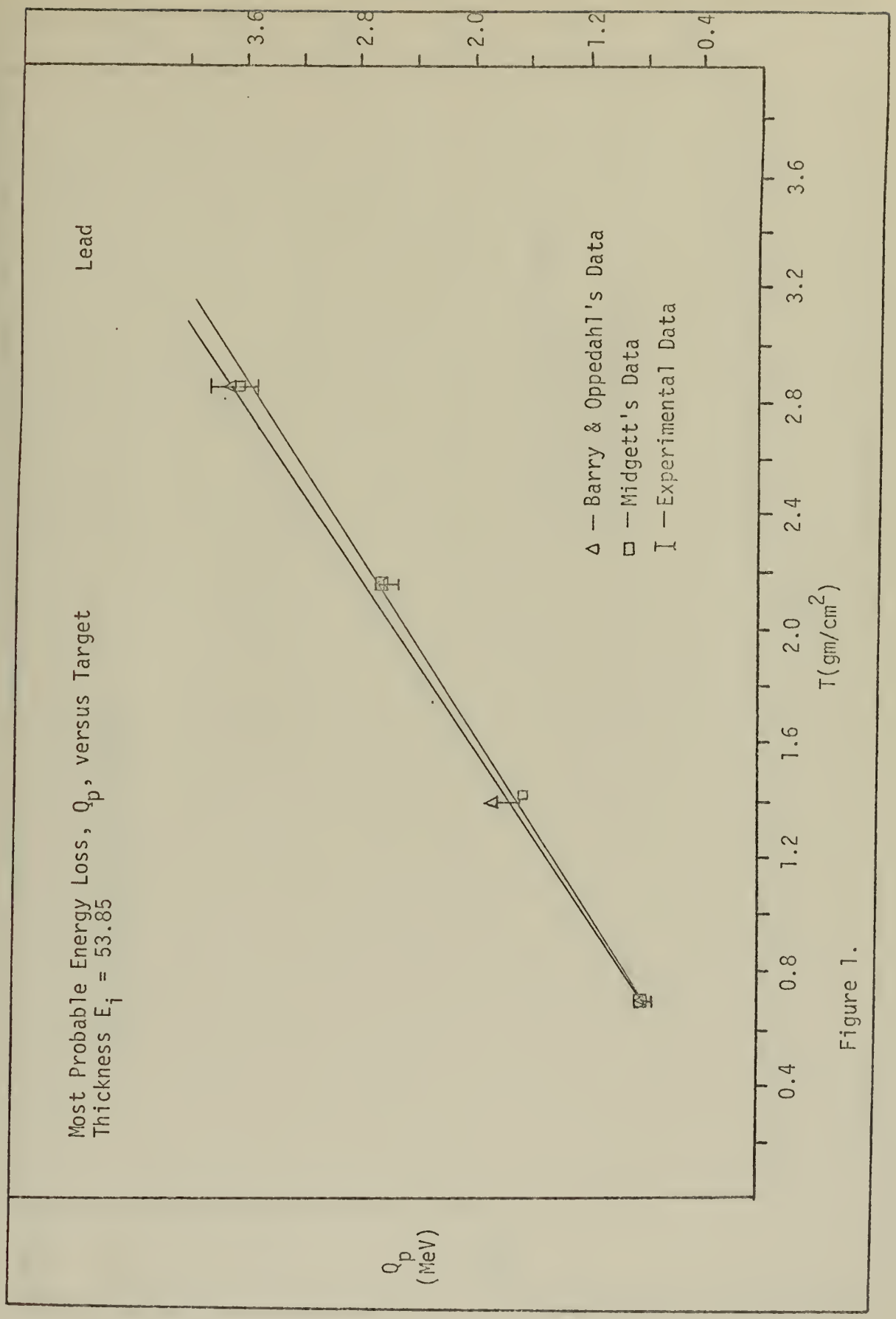
Most Probable Energy Loss, Q_p, versus Target Thickness E_i = 53.85
Lead
Δ — Barry & Oppedahl's Data
□ — Midgett's Data
I — Experimental Data
Q_p (MeV)
T(gm/cm^2)
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
0.4
1.2
2.0
2.8
3.6

Figure 1.

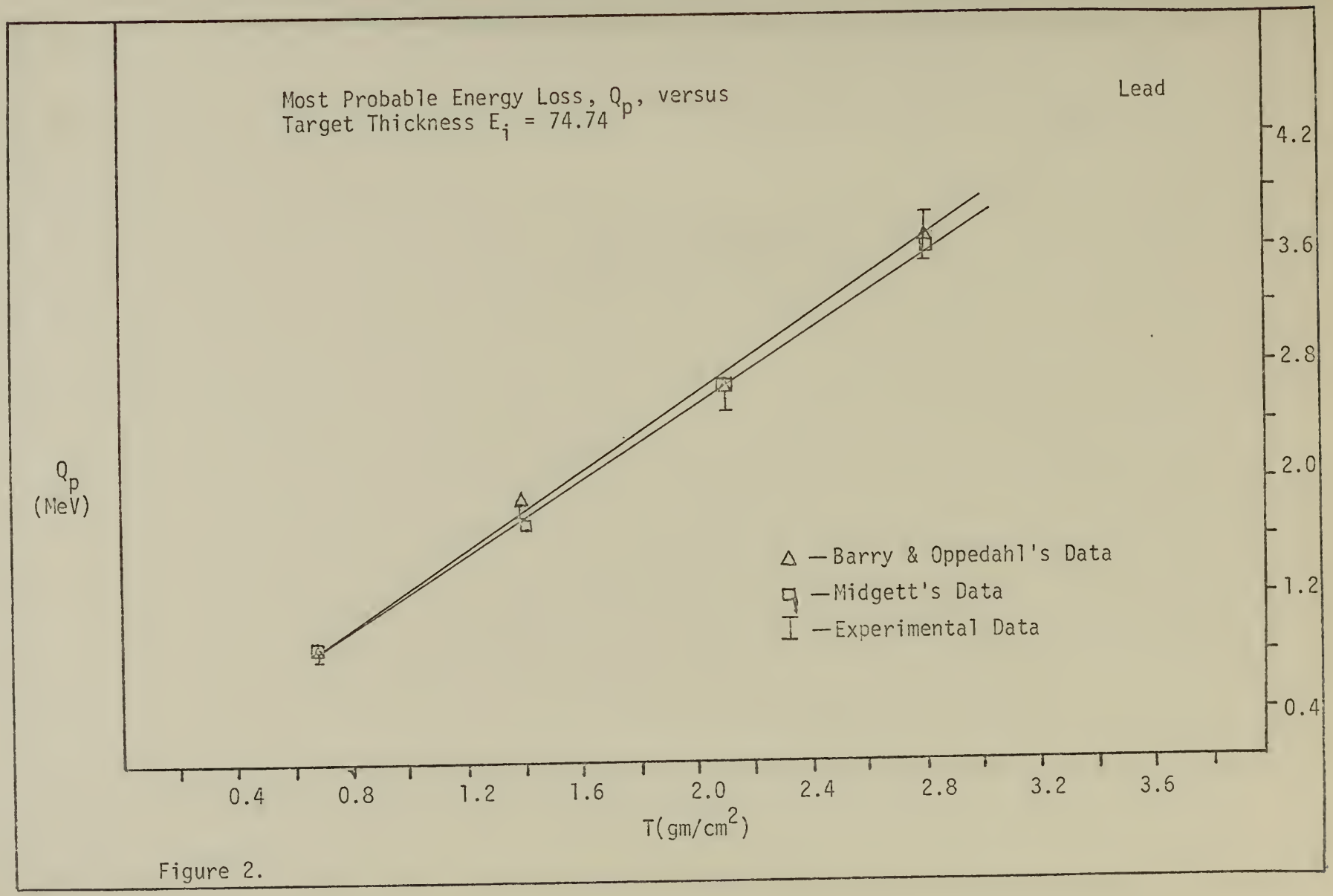
Most Probable Energy Loss, Q_p, versus
Target Thickness E_i = 74.74
Lead
Q_p (MeV)
4.2
3.6
2.8
2.0
1.2
0.4
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
T(gm/cm^2)
△ —Barry & Oppedahl's Data
—Midgett's Data
—Experimental Data

Figure 2.

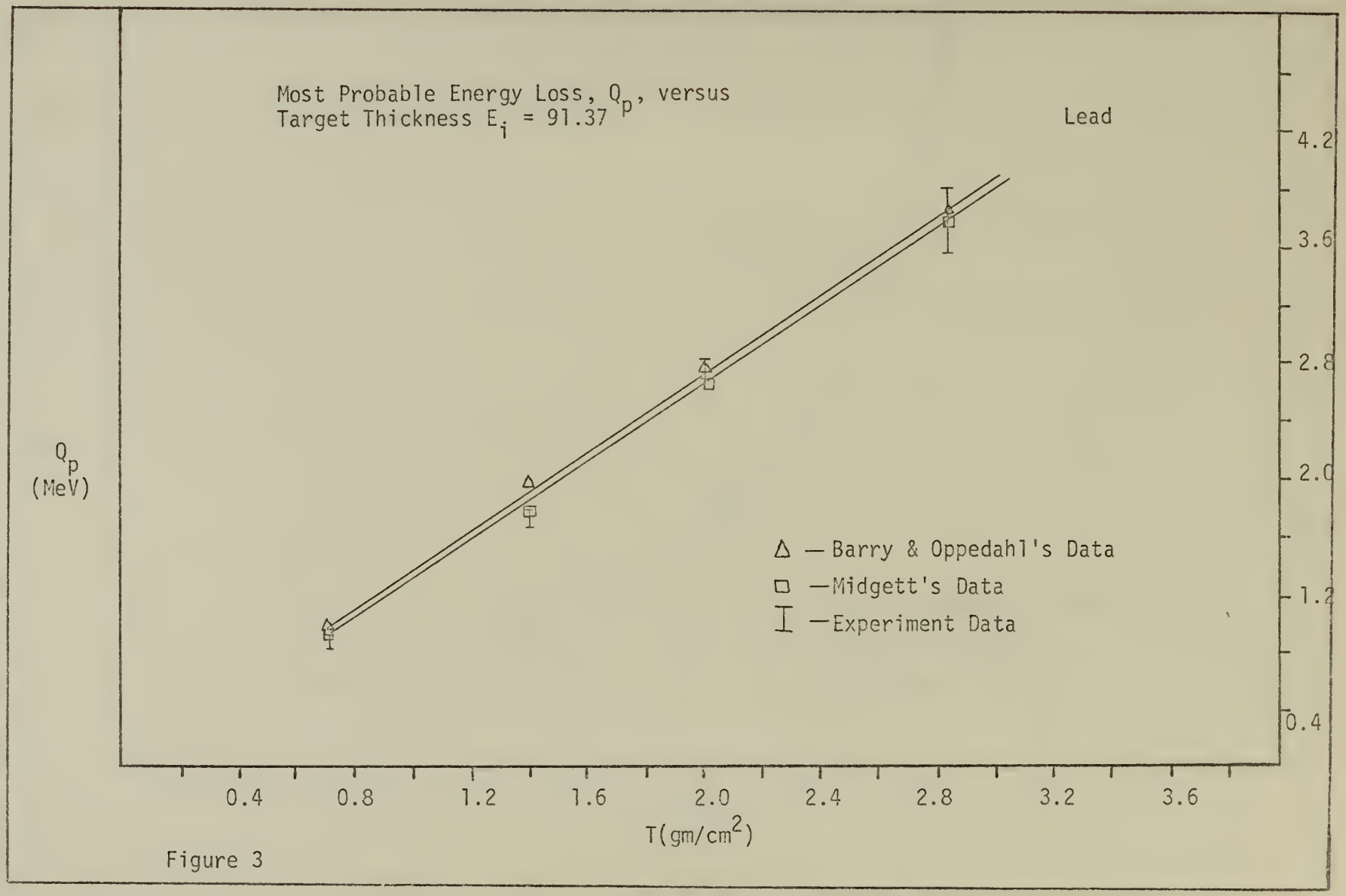
Most Probable Energy Loss, Q_p, versus
Target Thickness E_i = 91.37
Lead
Q_p (MeV)
4.2
3.6
2.8
2.0
1.2
0.4
Δ — Barry & Oppedahl's Data
□ —Midgett's Data
I —Experiment Data
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
T(gm/cm^2)

Figure 3

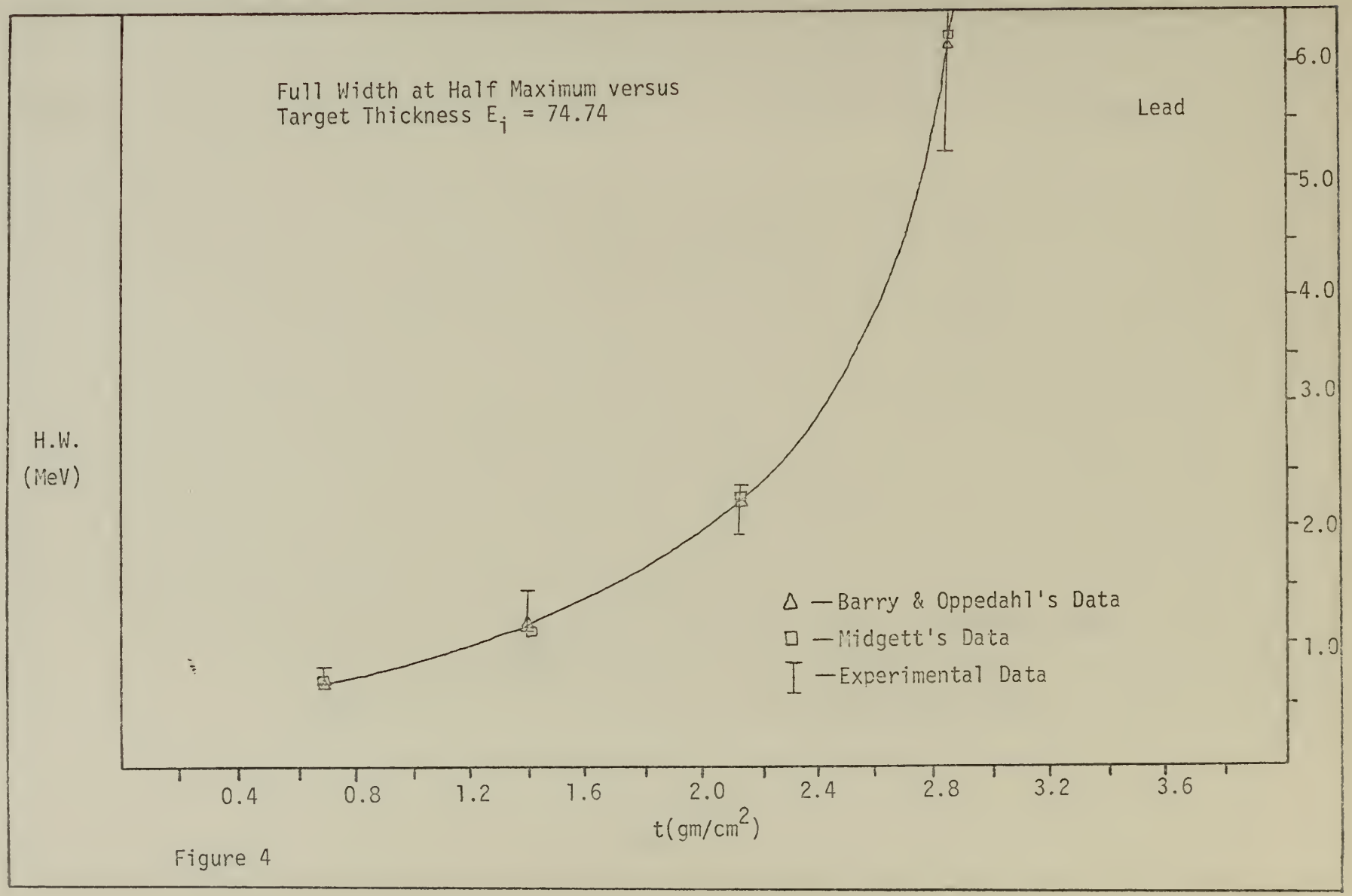
Full Width at Half Maximum versus
Target Thickness E_i = 74.74
Lead
H.W.
(MeV)
6.0
5.0
4.0
3.0
2.0
1.0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
t(gm/cm^2)
Δ —Barry & Oppedahl's Data
□ —Midgett's Data
I —Experimental Data

Figure 4

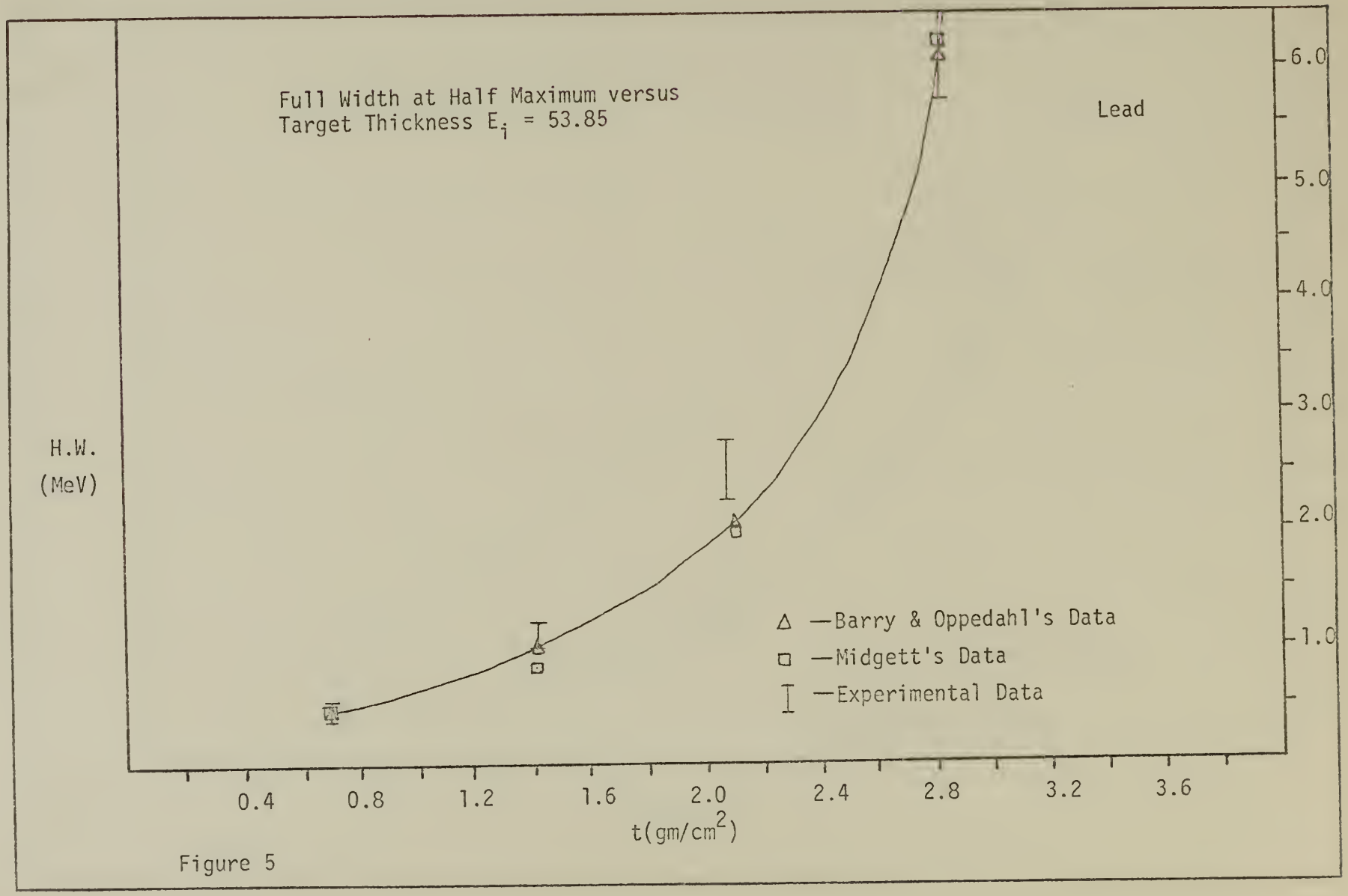
Full Width at Half Maximum versus
Target Thickness E_i = 53.85
Lead
H.W.
(MeV)
6.0
5.0
4.0
3.0
2.0
1.0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
t(gm/cm^2)
△ —Barry & Oppedahl's Data
□ —Midgett's Data
I —Experimental Data

Figure 5

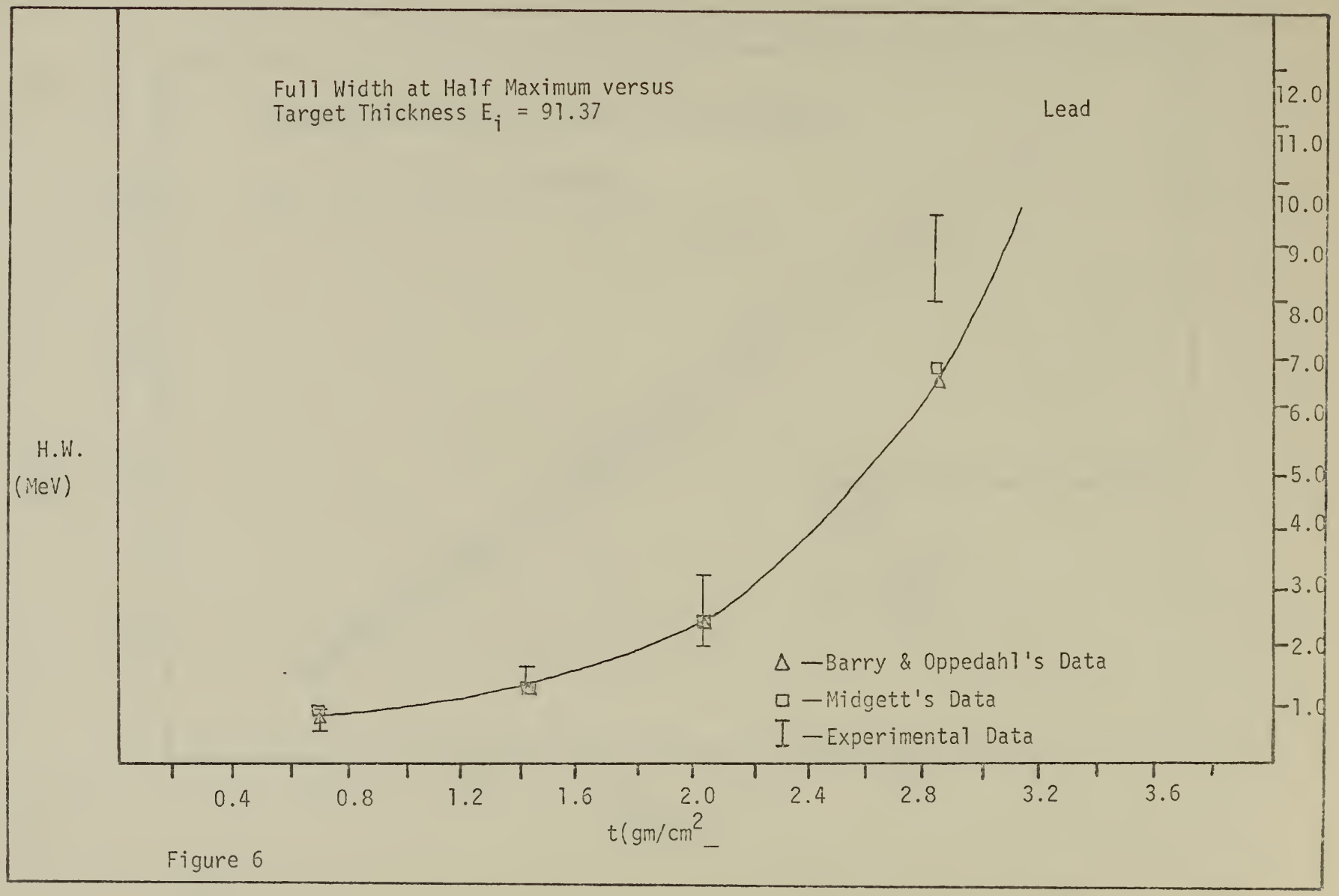
Full Width at Half Maximum versus
Target Thickness E_i = 91.37
Lead
H.W.
(MeV)
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
t(gm/cm^2)
△ —Barry & Oppedahl's Data
□ —Midgett's Data
I —Experimental Data

Figure 6

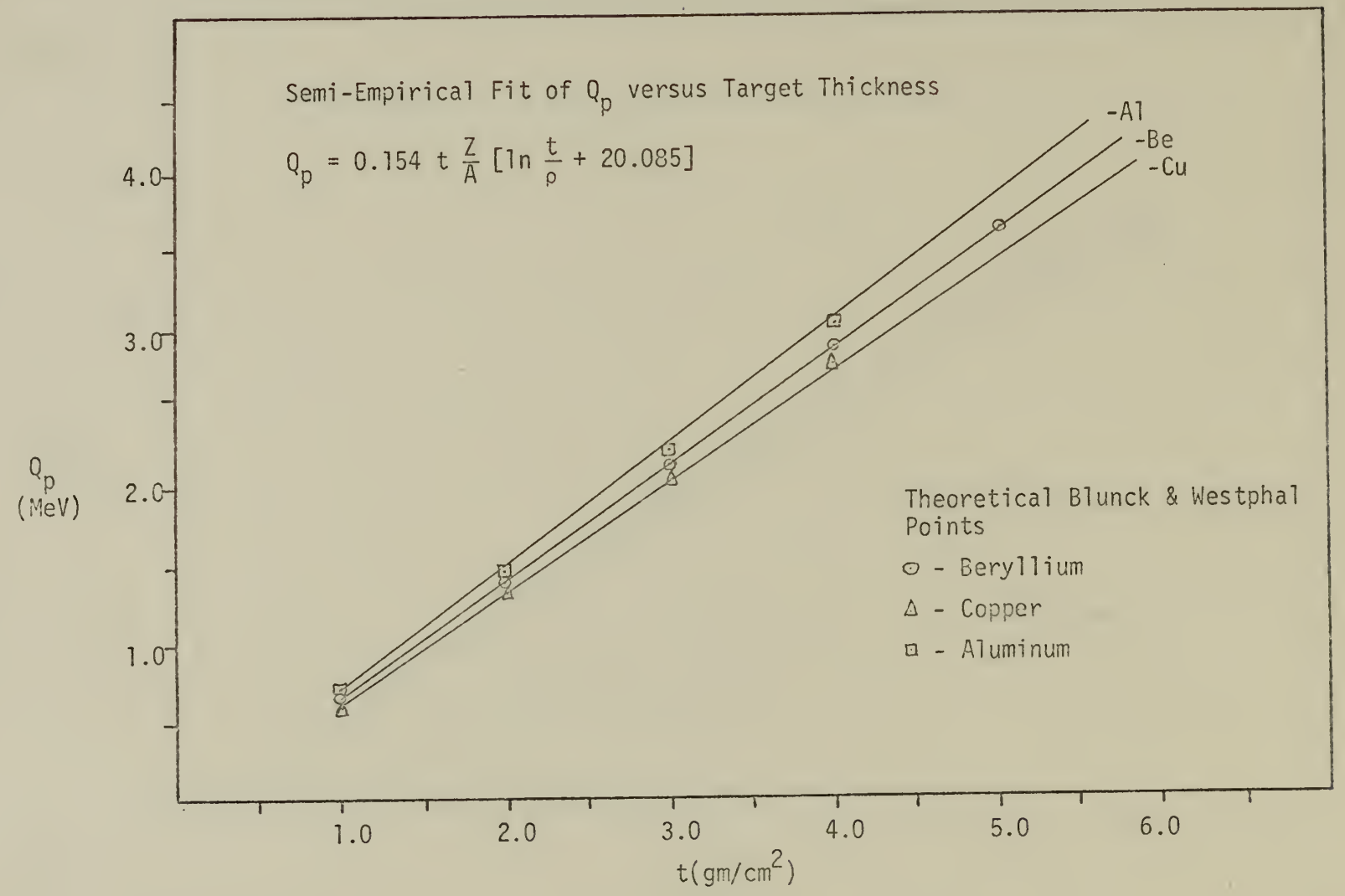
Semi-Empirical Fit of Q_p versus Target Thickness
$Q_p = 0.154\ t\ \frac{Z}{A}\ [\ln \frac{t}{\rho} + 20.085]$
-Al
-Be
-Cu
Theoretical Blunck & Westphal Points
⊙ - Beryllium
△ - Copper
□ - Aluminum
Q_p (MeV)
4.0
3.0
2.0
1.0
1.0
2.0
3.0
4.0
5.0
6.0
$t(gm/cm^2)$

Figure 7

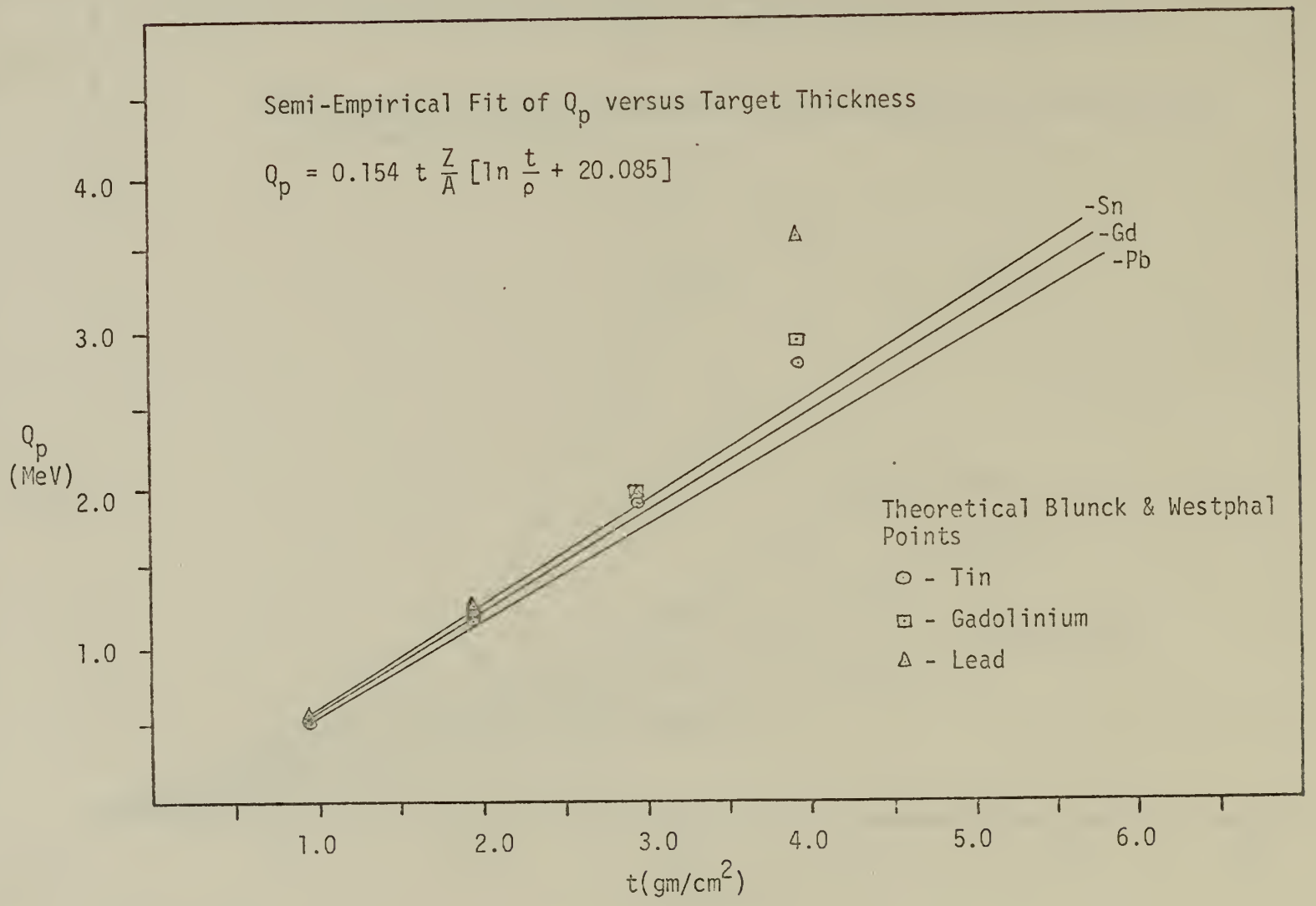
Semi-Empirical Fit of Q_p versus Target Thickness
$Q_p = 0.154\ t\ \frac{Z}{A}\ [\ln \frac{t}{\rho} + 20.085]$
-Sn
-Gd
-Pb
Theoretical Blunck & Westphal Points
○ - Tin
⊡ - Gadolinium
△ - Lead
Q_p (MeV)
4.0
3.0
2.0
1.0
1.0
2.0
3.0
4.0
5.0
6.0
t(gm/cm^2)

Figure 8

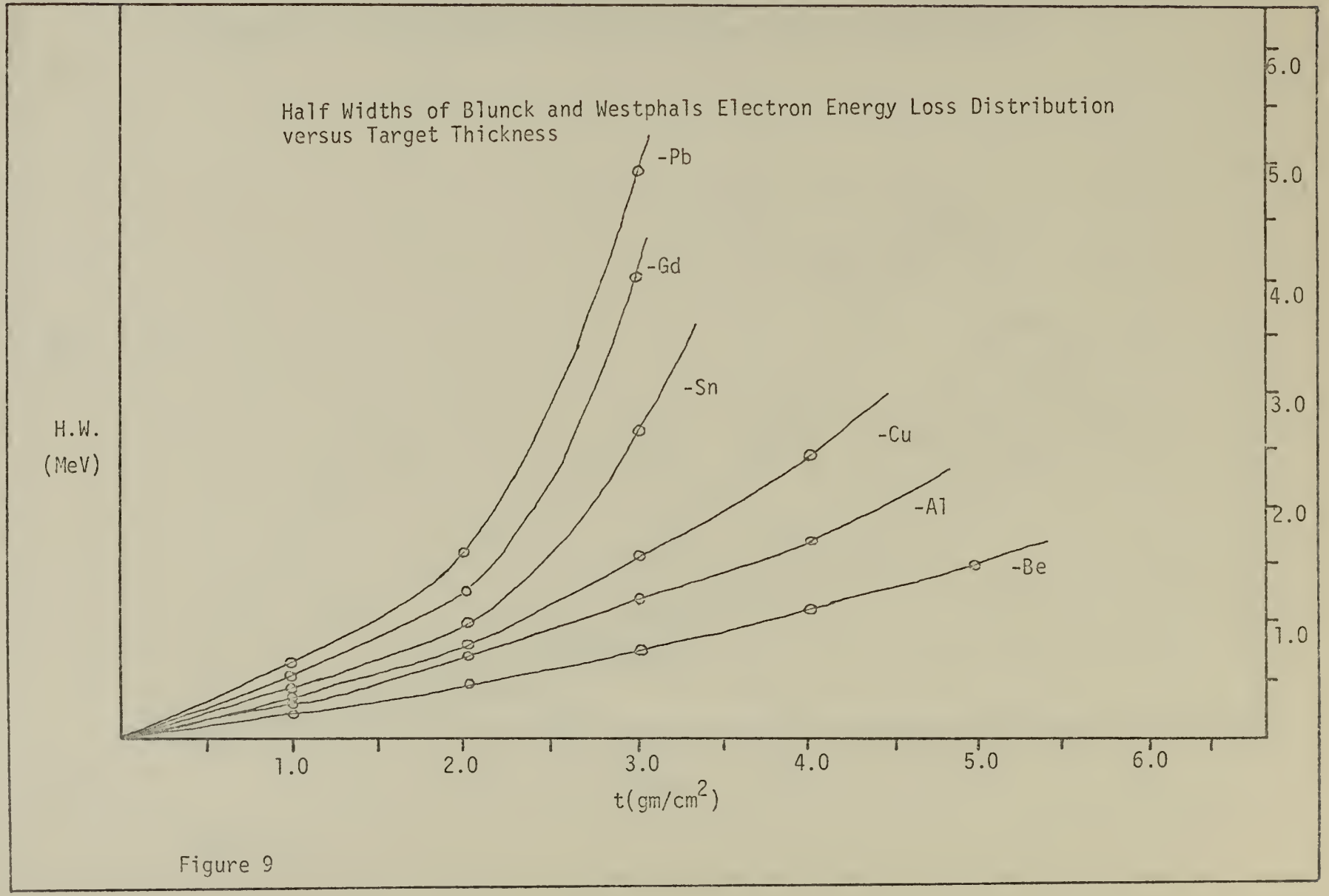
Half Widths of Blunck and Westphals Electron Energy Loss Distribution
versus Target Thickness
H.W.
(MeV)
-Pb
-Gd
-Sn
-Cu
-Al
-Be
1.0
2.0
3.0
4.0
5.0
6.0
t(gm/cm^2)

Figure 9

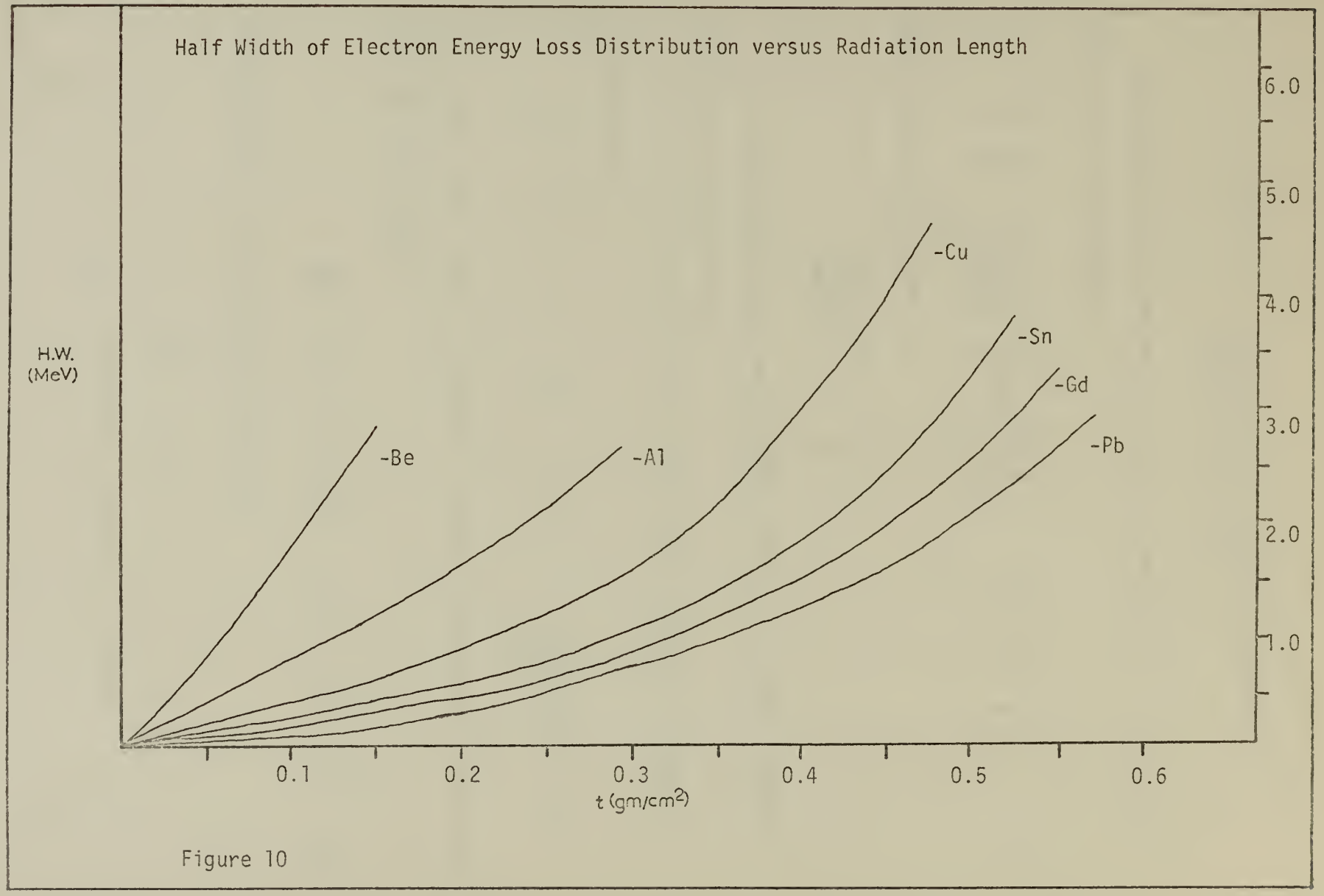
Half Width of Electron Energy Loss Distribution versus Radiation Length
H.W.
(MeV)
-Be
-Al
-Cu
-Sn
-Gd
-Pb
6.0
5.0
4.0
3.0
2.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
t (gm/cm²)

Figure 10

APPENDIX C - DERIVATION OF NUMERICAL CONSTANT IN EQUATION FOR LAMBDA

Sternheimer [11] gives for the correct expression for the average energy loss of electrons:

$$\frac{1}{\rho}\frac{dE}{dX} = \frac{2\pi n Z e^4}{mc^2\beta^2 A\rho}\left[\ln\frac{mc^2\beta^2 T}{(1-\beta^2) I^2} + \frac{9}{8} - \beta^2 - \delta\right] \qquad (1)$$

where the factor in front is:

$$\frac{2\pi N Z e^4}{mc^2\beta^2 A} = \frac{a}{\rho} \quad .$$

T is the maximum transferable energy, and for electrons impact on electrons, is equal to $E_i/2$.

Landau's expression for the average energy loss due to distant collisions is:

$$\int \varepsilon\, \omega(\varepsilon)\, d\varepsilon = a \ln\frac{\varepsilon_1}{\varepsilon'} \quad , \text{ where } \ln\varepsilon' = \frac{\ln(1-\beta^2) I^2}{2mc^2\beta^2} + \beta^2 \, .$$

Substituting this into (1) gives the following expression for the energy loss per path length:

$$\frac{dE}{dX} = \overline{K} = a\left[\ln\frac{E_i}{4} - \ln\varepsilon' + \frac{9}{8} - \delta\right] \, . \qquad (2)$$

The parameter λ was defined by Landau as:

$$\lambda = \frac{Q}{aX} - \ln aX + \ln\varepsilon' - 1 + C_B + \delta \, .$$

C_B = Bournelli's constant = 0.577

By eliminating the term $\ln \varepsilon'$ in λ by the expression for $\overline{K}$, equation (2) above, the expression for λ becomes:

$$\lambda = \frac{Q}{aX} - \frac{\overline{K}}{a} + \ln \frac{E_i}{aX} - \ln 4 + C_B + \frac{9}{8} - 1 \quad .$$

The last four constants add to give a numerical value of 0.686. Therefore, the expression for λ becomes:

$$\lambda = \frac{Q}{aX} - \frac{\overline{K}}{a} + \ln \frac{E_i}{aX} - 0.686 \quad .$$

Previous calculations involving energy loss of electrons has used the constant 1.116 in the equation for lambda. However, calculations in this paper used the constant 0.686, where the density effect has been taken into account. The differences in the values of these constants can be significant for values of lambda of the order of 1.0. These values for lambda are found for values of Q near the peak of the energy loss distribution.

APPENDIX D - DERIVATION OF SCREENING CORRECTION CONSTANT "F"

In calculating the probabilities for radiation processes, it is necessary to take into account the screening of the Coulomb field by outer atomic electrons particularly when the average impact parameter of the electron is of the same order of magnitude as the atomic radius. This screening effect must be taken into account when computing the energy loss of electrons.

An electron of initial energy E_i will loose energy by the emission of radiation as it passes near a heavy nucleus (bremsstrahlung). The probability of this electron to emit a photon with energy between E' and E'+dE' after traversing a thickness of $dT(gm/cm^2)$ is given in an article by Bethe and Heitler [12] as:

$$\Phi_R(E_i E')dE'dT = 4\,\alpha\,\frac{N}{A}\,Z^2\,r_e^2\,\frac{dE'}{E'}\,F(E_i\,\upsilon)$$

where:

α = fine structure constant = 1/137,

r_e = electron radius = e^2/mc^2,

$F(E_i\upsilon)$ = correction constant taking into account complete screening by other atoms.

The parameter υ, is defined in Rossi [15], as:

$$\upsilon = \frac{E'}{E_i + mc^2}$$

For electrons the assumption that $mc^2 << E_i$ can be made, which in turn will make:

$$\upsilon = \frac{E'}{E_i} \quad .$$

Rossi also defines the parameter:

$$\gamma = \frac{100\ mc^2}{E_i + mc^2} \cdot \frac{\upsilon}{1-\upsilon} Z^{-1/3}$$

where:

$$\gamma \simeq \frac{r_{atom}}{r_{eff}} \quad ;$$

r_{eff} is the effective impact parameter. For $\gamma << 1$, the effective radius much greater than the radius of the atom, complete screening is defined. The basic assumption of the Blunck and Westphal theory is that the energy loss is small compared to the initial energy of the electron. Therefore, $E' << E_i$ and $\gamma << 1$ and $\upsilon << 1$.

For complete screening, Bethe and Heitler give for the screening correction constant:

$$F(E_i\upsilon) = [1 + (1 - \upsilon)^2 - \frac{2}{3}(1 - \upsilon)] \ln 183\ Z^{-1/3} + \frac{1}{9}(1 - \upsilon)$$

For $\upsilon << 1$ the function $F(E_i\upsilon)$ becomes:

$$F = \frac{4}{3} \ln 183\ Z^{-1/3} + \frac{1}{9} \quad .$$

APPENDIX E - DERIVATION OF RADIATION LENGTH

The average radiation loss of an electron of energy E_i per gm/cm^2 material is: (See Rossi [15], Bethe-Heitler [12]):

$$\frac{dq}{dT} = \int_0^{E_i} \Phi_R(E_i E')\, E' dE' = 4\, \alpha \frac{N}{A} Z^2\, r_e^2\, F\, E_i$$

$$\alpha_R \text{ is defined as } = 4\, \alpha \frac{N}{A} Z^2\, r_e^2\, F \quad (\text{gm/cm}^2)^{-1} \quad ,$$

where F is a screening correction constant to take into account the screening of the electrons by the Coulomb field of orbit electrons. Defining α_R as above, the average energy loss per gm/cm^2 material can be written,

$$\frac{dE}{dT} = -\alpha_R E$$

Therefore,

$$E = E_o\, e^{-\alpha_R T}$$

By defining the radiation length as that length where the electron has lost one half of its energy,

$$\frac{1}{2} E_i = E_i\, e^{-\alpha_R T_R}$$

and

$$\frac{1}{T_R} = \frac{\alpha_R}{\ln 2}$$

The above definition for radiation length differs from that of the other authors where the radiation length is that length of material in which an electron has had its energy reduced to 1/e of the initial energy.

Putting numerical values into α_R gives:

$$\alpha_R = 1.38 \times 10^{-3} \frac{Z^2}{A} F \quad (gm/cm^2)^{-1} \quad .$$

APPENDIX F - REMOVAL OF THE DIVERGENCE OF THE INTEGRAL

The expression for Λ_γ has been defined as:

$$\Lambda_\gamma = \frac{\lambda - \lambda_\gamma}{\sqrt{b^2 + \gamma_\gamma^2}}$$

Therefore, the probability of energy loss Q is:

$$W(Q)dQ = \frac{aX}{E_i}^z \frac{1}{2}^{z/2} \frac{1}{\Gamma(z)} \sum_\gamma \frac{C_\gamma \gamma_\gamma \, e^{-\Lambda_\gamma^2}}{\sqrt{b^2 + \gamma_\gamma^2}^{1-z}} \int_0^{\eta_{max}} \eta^{z-1} \, e^{\sqrt{2}\Lambda_\gamma \eta - \frac{\eta^2}{2}} \, d\eta \, d\lambda \quad .$$

For $z < 1$ the integral diverges when calculated numerically. The actual integral is not singular, so a series expansion is taken; first order expansion gives the following:

$$\int_0^{\eta_m} \frac{1}{\eta^{1-z}} \, d\eta = \frac{\eta^2}{z}\Big|_0^{\eta_m} = \frac{\eta_m^2}{z} \quad \text{for } z \neq 0 \quad .$$

Integration by parts allows the integral to obtain a form suitable for handling by standard numerical routines.

$$T = \int_0^{\eta_m} \eta^{z-1} \, e^{\sqrt{2}\Lambda_\gamma \eta - \frac{\eta^2}{2}} \, d = \int_0^{\eta_m} \eta^{z-1} \, e^{\Phi(\eta)} \, d\eta \qquad (1)$$

$$T = \frac{\eta^2}{z} \, e^{\Phi(\eta)} \Big|_0^{\eta_{max}} - \int_0^{\eta_m} \frac{\eta^z}{z} \, e^{\Phi(\eta)} \, \Phi'(\eta) \, d\eta \qquad (2)$$

finally,

$$T = \frac{1}{z} \, \eta_m^z \exp\left[\sqrt{2} \, \Lambda_\gamma \eta_m - \frac{\eta_m^2}{2}\right] - \int_0^{\eta_m} \frac{\eta^z}{z} \, e^{\Phi(\eta)} \, (\sqrt{2}\Lambda_\gamma - \eta) \, d\eta$$

APPENDIX G - CORRELATION OF COMPUTER SYMBOLOGY AND EQUATION SYMBOLS

EO = E_i	energy of incoming electrons in MeV.
Z = Z	atomic number of target material.
A = A	mass number of target material.
T = T	target thickness in gm/cm^2.
BSQ = β^2	$(\text{electron speed}/c)^2$.
ALPHA = $\tilde{a}$	basic cross section constant.
V = F	correction constant taking into account complete screening in radiation loss.
AR = α_R	inverse radiation length times ln 2 in $(gm/cm^2)^{-1}$.
TAU = τ	target thickness in radiation length times ln 2.
Q = Q	energy loss of electrons in MeV.
RION = I(Z)	Bloch formula for average ionization potential.
X = X	variable for evaluation of density correction.
DELTA = δ	density correction according to Sternheimer.
BBARK = $\tilde{\overline{K}}$	average total energy loss $(MeV/gm/cm^2)$.
LAMBDA = λ	Landau lambda for energy loss distribution.
D(I) = C_γ	constants for superposition of W(Q) from Gaussians.
G(I) = γ_γ	
R(I) = λ_γ	
QBAR = $\overline{Q}$	average energy loss in target (MeV).
B = b	correction constant for second order term in resonance part of ionization loss.
CAPPI = Λ_γ	variable in Landau function.

ETAM = η_m upper limit of integration variable which is that portion of Q lost by radiation.

PSI = $\Psi(\eta)$ this term is defined for convenience in evaluation of some of the exponentials which occur in the equations.

FGAMMA = $\frac{1}{\tau}(\eta_m)\ e^{(\Psi(\eta_m) - \Lambda_\gamma^2)}$

FCT = $\frac{1}{\tau}\eta^\tau\ (\sqrt{2}\,\Lambda - \eta)\ e^{(\Psi(\eta) - \Lambda^2)}$

WQT = $\frac{1}{\tilde{a}\tilde{T}}\ (\frac{\tilde{a}\tilde{T}}{E_i})^\tau\ (\frac{1}{2})^{\tau/2}\ \frac{1}{\Gamma(\tau)}$

CONSTT = $\frac{C_\gamma \gamma_\gamma}{(b^2 + \gamma_\gamma^2)^{1-\tau}}$

```
      EXTERNAL FCT
      COMMON TAU,CAPP1,PPSI
      REAL LAMBDA
      REAL LAMBD
      REAL K,M
      DIMENSION WQ(4),CCC(4),FGAMMA(4)
      DIMENSION WQT(4),WQTT(4),CONSTT(4)
      DIMENSION D(4),G(4),R(4),CAPP(4),ETA(4)
      DIMENSION E00(12),COUNT(12),BIN(200)

READ IN THE CONSTANTS WHICH WILL FIT THE ENERGY LOSS
DISTRIBUTION TO A SUM OF FOUR GAUSSIANS.
      DATA D(1),D(2),D(3),D(4)/0.174,0.058,0.019,0.007/,G(1)
     1,G(2),G(3),G(4)/1.8,2.0,3.0,5.0/,R(1),R(2),R(3),R(4)/
     10.0,3.0,6.5,11.0/

THE NEXT THREE PARAMETERS GIVE THE STARTING VALUE FOR THE
ENERGY LOSS DISTRIBUTION (QS), THE WIDTH OF THE BINS FOR
 THE FINAL DISTRIBUTION (DELQ), AND THE WIDTH OF THE BINS
OF THE INITIAL DISTRIBUTION (DELQS).
      QS=1.7
      DELQ=.05
      DELQS=0.100001

"COUNTM" IS A NORMALIZING FACTOR FOR THE INITIAL ENERGY
DISTRIBUTION
      COUNTM=795.
      L=1

READ IN THE TARGET PARAMETERS AND DENSITY CORRECTION
CONSTANTS
      READ(5,21)  (E00(J),COUNT(J),J=1,12)
      READ(5,20) Z,A,T,X1,K,M,C
        E0=INCIDENT ELECTRON ENERGY
-       Z=ATOMIC NUMBER OF TARGET
        A=ATOMIC WEIGHT OF TARGET
        T=THICKNESS OF TARGET IN GM/CM2
        X1,K,M,C ARE THE STERNHEIMER DENSITY CORRECTION
        CONSTANTS.
 20 FORMAT(3F10.5,4F7.4)
 21 FORMAT (6F10.5)

 ZERO ALL THE BINS
      DO 600  N=1,200
600 BIN(N)=0.0

 INITIATE INTRODUCTION OF FIRST ENERGY BIN FOR THE INITIAL
 DISTRIBUTION
      DO 700 J=1,12
      E0=E00(J)
      N=L
      WRITE(6,25)E0,Z,A,T,X1,K,M,C
 25 FORMAT('  TARGET PARAMETERS AND DENSITY CORRECTION
     1CONSTANTS',//,' E0='F6.2,3X,' Z='F6.2,3X,' A=' F8.4,
     13X,' T='F9.5,/,' X1='F5.2,3X,' K='F6.3,3X,' M='F8.4,
     13X,' C='F8.4,//)

 BSQUAR=BSQ  IS EQUAL TO THE SQUARE OF THE ELECTRONS SPEED
 DIVIDED BY THE SQUARE OF THE SPEED OF LIGHT.
      BSQ=1.0-0.25/E00(J)**2

 ALPH=ALPHA  IS THE BASIC CROSS SECTION CONSTANT IN UNITS
 OF MEV/GM/CM SQUARED.
      ALPHA=(0.154*Z)/(BSQ*A)

 FUNC(Z)= IS THE CORRECTION CONSTANT TAKING INTO ACCOUNT
 COMPLETE SCREENING IN RADIATION LOSS.  V= FUNC(Z)
      V=(4./3.)*(ALOG(183./Z**(1./3.)))+1./9.
```

```
XX=X  IS THE VARIABLE USED FOR EVALUATION OF DENSITY
CORRECTION.
     X=ALOG10(E00(J)/.511)

    AR=ARAD(Z,A,V)

TAU IS THE TARGET THICKNESS IN RADIATION LENGTH TIMES LN2
    TAU = T*AR

RION  IS THE BLOCH FORMULA FOR AVERAGE IONIZATION POTENT.
    RION = 13.5E-6*Z

THE NEXT SECTION SELECTS THE CORRECT STERNHEIMER COR-
RECTION CONSTANT FOR THE TARGET BEING USED.. DELTA IS THE
STERNHEIMER CORRECTION TERM.
    IF(X.LT.X1) GO TO 40
    DELTA2 = 4.606*X+C
    DELTA = DELTA2
    GO TO 50
 40 DELTA1 = 4.606*X+C+K*(X1-X)**M
    DELTA = DELTA1
 50 WRITE(6,60) DELTA
 60 FORMAT(' DELTA='F12.6)


    WRITE(6,10) BSQ,ALPHA,V,AR,TAU,RION,X
 10 FORMAT(//,' BSQ='F12.9,/,' ALPHA='F12.9,/,' V='F12.9,/
   1F12.9,/,' TAU='F12.9,/,' RION='F12.9,/,' X='F12.9,//)
    BBARK = BARK(ALPHA,E0,BSQ,RION,DELTA)
    WRITE(6,70)BBARK
 70 FORMAT(' AVERAGE TOTAL ENERGY LOSS ='F12.7,//)
QBAR IS THE AVERAGE ENERGY LOSS IN THE TARGET
    QBAR=T*BBARK

    B=BB(Z,ALPHA,T,QBAR)
    WRITE(6,100)QBAR,B
100 FORMAT(' QBAR='F12.7,' B='F12.7,//)
 "Q"  IS THE ENERGY LOSS OF ELECTRONS IN MEV.
    Q=QS

220 LAMBDA=LAMBD(Q,ALPHA,T,E0,BBARK)
    AWQ=0.0
    DO 105 I=1,4
    DD=D(I)
    GG=G(I)
    RR=R(I)
    CALL CAPLAM(P,LAMBDA,RR,GG,B,CCAPP)
    CAPP(I)=CCAPP
    CAPP1=CAPP(I)

THIS NEXT SECTION DOES THE INTEGRATION OF THE FUNCTION
F(ETA).  CC=ETAM=ETAMAX  IS  THE UPPER LIMIT OF INTEGRA-
TION.  "NN"  IS A FACTOR WHICH DIVIDES THE FINAL DISTRI-
BUTION INTO NN PARTS IN ORDER TO GET MORE ITERATIONS FOR
THE FUNCTION BEING INTEGRATED.
    CCC(I)=ETAM(Q,B,GG,ALPHA,T)
    CC=CCC(I)
    ETAMAX=CC
    SUM=0.0
    XL=0.0
    NN=10.
    XU=ETAMAX/NN
    DELTA=XU
QG10 IS A TEN POINT GAUSSIAN QUADRATURE INTEGRATION
ROUTINE FOUND IN THE NPS LIBRARY OF ROUTINES.
310 CALL QG10(XL,XU,FCT,Y)
    SUM=SUM+Y
    XL=XU
    XU=XU+DELTA
    IF(XU.GT.ETAMAX) GO TO 30
    GO TO 310
```

```
 30 PSI=PSII(CAPP1,CC)
    FGAMMA(I)=FGAMA(TAU,CC,CAPP1,PSI)
    WQT(I)=WQWQ(ALPHA,T,EO,TAU)
    CONSTT(I)=CONST(DD,GG,B,TAU)
    WQ(I)=WQT(I)*CONSTT(I)*(FGAMMA(I)-SUM)

 AWQ IS THE VALUE FOR W(Q)DQ ASSUMING A MONOENERGETIC
 SOURCE OF ELECTRONS, AND THE BINS REPRESENT W(Q)DQ FOR
 THE UNFOLDED THEORY.
    AWQ=WQ(I)+AWQ
105 CONTINUE
    BIN(N)=BIN(N)+AWQ*(COUNT(J)/COUNTM)
    WRITE(6,650) Q
650 FORMAT(' Q='F6.3)
    WRITE(6,610)  (N,BIN(N))
610 FORMAT(' BIN(',I3,' )=',F12.9)
    N=N+1
    Q=Q+DELQ
    IF(5.5-Q) 500,220,220
500 L=L+DELQS/DELQ
700 CONTINUE
    STOP
    END


    REAL FUNCTION ARAD(Z,A,Y)
 ARAD=AR  IS THE INVERSE RADIATION LENGTH TIMES THE LN 2.
 UNITS IN (GM/CM) TO THE MINUS ONE.
    X = 1.38E-3*Z**2
    W = X*Y
    ARAD = W/A
    RETURN
    END


    REAL FUNCTION BARK(ALPHA,EO,BSQ,RION,DELTA)
 BBARK=BARK IS THE AVERAGE TOTAL ENERGY LOSS
    A = ALOG((.511*EO*BSQ)/(2.0*(1.0-BSQ)*RION**2))
    B = A+9.0/8.0-BSQ
    C = B-DELTA
    BARK = ALPHA*C
    RETURN
    END


    REAL FUNCTION LAMBD(Q,ALPHA,T,EO,BBARK)
 LAMBD IS THE LANDAU LAMBDA AS CORRECTED BY STERNHEIMER.
    A=Q/(ALPHA*T)
    B=A-BBARK/ALPHA
    C=B+ALOG(EO/(ALPHA*T))
    LAMBD=C-0.685
    RETURN
    END


    REAL FUNCTION BB(Z,ALPHA,T,QBAR)
 B=BB  IS THE CORRECTION CONSTANT FOR SECOND ORDER TERM
 IN RESONANCE PART OF THE IONIZATION LOSS.
    A=Z**(2.0/3.0)
    B=A/(ALPHA*T)
    C=SQRT(20.0E-6*QBAR)
    BB=B*C
    RETURN
    END
```

```
      SUBROUTINE CAPLAM(P,Y,R,G,B,CCAPP)
CAPLAM  IS THE VARIABLE IN THE LANDAU FUNCTION
      P=Y-R
      CCAPP=P/SQRT(B**2+G**2)
      RETURN
      END

      REAL FUNCTION ETAM(Q,B,G,ALPHA,T)
ETA MAX IS THE UPPER LIMIT OF INTEGRATION VARIABLE WHICH
IS THAT PORTION OF Q LOST BY RADIATION.
      A=SQRT(2.0)
      Y=SQRT(B**2+G**2)
      C=A/Y
      ETAM=C*Q/(ALPHA*T)
      RETURN
      END

      REAL FUNCTION FCT(ETA)
THE FUNCTION F(ETA) IS THE FUNCTION TO BE INTEGRATED FROM
0 TO SOME VALUE ETA(MAX).
      COMMON TAU,CAPP1,PPSI
      PPSI=PSII(CAPP1,ETA)
      A=1.0/TAU
      B=A*(ETA**TAU)
      C=(SQRT(2.0)*CAPP1)-ETA
      D=PPSI-(CAPP1**2)
      IF(ABS(D).GT.150.0) GO TO 10
      E=EXP(D)
      GO TO 20
10    E=0.0
20    FCT=B*C*E
      RETURN
      END

      REAL FUNCTION FGAMA(TAU,ETAM,CAPP1,PSI)
      A=1.0/TAU
      B=A*(ETAM**TAU)
      C=EXP(PSI-CAPP1**2)
      FGAMA=B*C
      RETURN
      END

      REAL FUNCTION WQWQ(ALPHA,T,E0,TAU)
THIS IS A CONSTANTS AND CONSISTS OF THE FIRST PART,
BEFORE THE SUM, IN  THE EQUATION FOR W(Q).
      A=1./(ALPHA*T)
      B=((ALPHA*T)/E0)**TAU
      C=A*B
      D=C*(1./2.)**(TAU/2.)
      E=1./GAMMA(TAU)
      WQWQ=D*E
      RETURN
      END

      REAL FUNCTION PSII(CAPP1,ETA)
      A=SQRT(2.0)*CAPP1*ETA
      B=(ETA**2)/2.0
      PSII=A-B
      RETURN
      END

      REAL FUNCTION CONST(D,G,B,TAU)
```

```
A=D*G
C=(B**2+G**2)**(1.0-TAU)
CONST=A/(SQRT(C))
RETURN
END
```

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13. ABSTRACT

A reformulation of the Blunck and Westphal theory of electron energy loss in metals was performed for the metals beryllium, aluminum, copper, tin, gadolinium, and lead. Comparison with previous theoretical calculations shows good agreement for the most probable energy loss and for the full widths at half maximum of the electron energy loss distributions. The computer program designed for these calculations is an improvement over previous programs in both computation time and simplicity.

A semi-empirical formula for the most probable energy loss was calculated. This formula agrees, within a few percent, with the most probable energy loss calculated according to the Blunck and Westphal theory except in the case of thick (>3 gm/cm^2) absorbers of heavy elements. The full width at half maximum for the energy loss distribution is presented in graphical form as functions of target thickness and atomic number. Both the most probable energy loss and the half widths were found to be only slightly dependent upon the initial electron energy.

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